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The Inductive Method in Teaching Continuity of Functions in Tunisian Secondary Education

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Abstract: This article aims to address the challenges involved in teaching the continuity of real functions in Tunisian secondary schools, a context marked by a tension between intuitive approaches and the formal (ε, δ) definition. Curriculum analysis reveals that the inductive method is a key element in terms of approach and reasoning. However, despite being prescribed as a tool for discovery and generalization based on specific cases, intended to build an intuitive definition that leads to the formal one; the inductive method often fails to ensure this transition. This study is based on a cross-analysis: on the one hand, a praxeological study (Anthropological Theory of Didactics, TAD) of curricula and textbooks reveals that induction is limited to intuitive technology, without explicit generalization or structured treatment of the negation of discontinuity. On the other hand, a questionnaire is filled out by 67 teachers, analyzed according to the Mathematical Knowledge for Teaching (MKT) model, shows that the majority of teachers prefer inductive approaches (61.5% to 66.1% depending on the items), but highlights perceived gaps in students' knowledge (generalization, approximation, graphical interpretation) and calls for epistemological training. The results confirm a disconnect between curricular intentions and stated practices, with incomplete realization of the inductive process in teaching resources and only partial mobilization by teachers. A number of avenues for reform are proposed: enriching textbook activities with explicit stages of generalization and logic, strengthening teacher training on the link between intuitive and formal registers and the meta-mathematical dimensions, and using digital tools to stimulate inductive experimentation.

Keywords: Induction, Continuity, Definition of ε-δ, Anthropological Theory of Teaching (TAD), Mathematical Knowledge for Teaching (MKT).

1. Introduction

The teaching of the continuity of a real function in Tunisian secondary schools is marked by a recurring tension between an intuitive approach (graphical, kinematic) and rigorous formalization (ε-δ definition). This tension has been the subject of research (El Bouazzaoui, 1988; Selmi, 2024), where students' spontaneous representations often clash with the abstractions of mathematical rigor, thus preventing the scientific conceptualization of the concept. Facing these difficulties, some education systems, such as those in France and Vietnam, have chosen to remove the formal definition from secondary school curricula, postponing this learning to higher education. However, this formalism is also identified as a major obstacle to the transition from secondary school to university. It is a source of difficulty that contributes to a high first-year university failure rate (Artigue, 1999; Bloch, 2000; Artigue, 2004; Ghedamsi, 2008; Sghaier, 2019). In this international context and since 2008, Tunisia

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has adopted a curricular approach that introduces the formalism of definitions of the continuity and limit concepts of a real function in terms of ε-δ for certain levels of secondary education (official mathematics programmes, 2008). These programmes prescribe a pedagogical approach in which concepts must be discovered by students through observation and experimentation. This prescription is based on a vision of induction, defined as a process of generalization from specific cases (Pólya, 1954), and established as the favorite approach to knowledge construction. Current research (Singh & Yadav, 2017; Hasibah et al., 2018; Misrom & al., 2020; Purnawasi, 2024) supports the use of the inductive method to promote active learning and conceptual understanding in mathematics at different levels of education. Applied to the concept of continuity, this method is designed as an essential mediation to develop students' intuition towards a 'zero-definition' (Lakotos, 1984), thus paving the way for subsequent formalization in ε-δ terms. However, despite this requirement, the inductive approach often seems to fail to enable the transition from intuition to formalization. The ε-δ definition is frequently perceived as a foreign and imposed object, rather than the result of reasonable generalization. This observation reveals a significant gap between curricular intentions, on the one hand, and actual implementation in classrooms on the other. It also highlights a lack of systematic research on the inductive approach in constructing the definition of continuity and on the professional knowledge teachers need to guide. Our research takes this observation as its starting point to ask the following central question: why does induction, despite being prescribed, often seem to fail to ensure the transition from the conceptual image (Tall & Vinner, 1981) to the formal definition (ε-δ) of continuity, and thus to alleviate the problems encountered in higher education?

The hypothesis we explore is that the problem does not lie in the absence of the inductive approach, but in its truncated or incomplete implementation within the teaching system. To test this hypothesis, our study adopts a dual analytical perspective, which is original in its combination. On the one hand, an analysis of resources (curricula and textbooks) is conducted through the framework of Anthropological Theory of Didactics (TAD) in order to examine how induction is integrated into praxeological organizations and the role assigned to it in the construction of knowledge. On the other hand, an analysis of teachers' knowledge is conducted using the Mathematical Knowledge for Teaching (MKT) model to identify the knowledge they use to interpret and implement this approach, as well as their perception of its usefulness and limitations.

By combining these two analyses, we aim to accurately identify the breaking points in the continuity learning process and propose ways to optimize the use of induction as a lever for building mathematical knowledge that is both intuitive and rigorous.

2. Theoretical framework

2.1. Inductive method and induction in mathematics: conceptual distinction

This study makes an essential distinction between two meanings of induction. On the one hand, mathematical induction (or reasoning by recurrence), which is a rigorous deductive method of proof, it is not the subject of this work. On the other hand, induction in mathematics is considered here as a method of discovery (Pólya, 1967), borrowed from the experimental sciences, aimed at generating conjectures from the observation of specific cases.

This epistemological distinction is not universally accepted. The famous mathematician H. Poincaré (1902), in Science and Hypothesis, considers reasoning by recurrence to be a form of induction. On the other hand, B. Russell (1905) (cited by Crocco et al., 2020) in his review of

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Science and Hypothesis, disputes the truly inductive nature of reasoning by recurrence. These divergent positions highlight the complexity of the issue.

For G. Pólya (1954), induction is defined as a process of generalization based on specific observations. It is a pillar of scientific and mathematical reasoning, not as a guarantee of truth, but as a driver of discovery. In Mathematics and Plausible Reasoning, Polya shows how inductive reasoning can be used to formulate plausible conjectures, the definitive validation of which subsequently requires deductive proof. Thus, in this research, the inductive method refers to the process that uses this reasoning to construct provisional results, the certainty of which can only be assured by subsequent deductive validation (the different types of mathematical reasoning are studied by Jeannotte (2015) in his doctoral thesis).

2.2.The role assigned to the inductive method in official curricula

The secondary school mathematics curricula (2008) give inductive reasoning an explicit and legitimate role, on a par with deductive reasoning. Induction has a three fold role: it is presented as a tool for discovery and exploration, an initial phase in learning a new concept, and a means of generating knowledge from observation and experimentation. The following excerpt is significant: “Students will develop their abilities to research, experiment, conjecture, or verify a result. Similarly, they will develop chains of inductive, deductive, reduction ad absurdum, or recursive reasoning” (Official Mathematics Curricula, 2008, p. 3). The prescribed structure is clear, induction must lead to a phase of rigorous validation (deductive proof). In the case of continuity, it is therefore supposed to prepare the ground for the synthesis and formalization of the ε-δ definition presented in the textbook. This role is summarized in the following table:

Table 1: The role assigned to the inductive method in official curricula

Aspect	Recommended role in programmes
Status	A fundamental and legitimate approach, just like deduction
Function	Tool for discovery, exploration and conjecture formulation
Timing	Initial phase of learning a new concept
Aim	Generating knowledge through observation and experimentation
Interconnection with other types of reasoning	Must lead to a rigorous validation phase (deductive proof)

This analysis of the programmes serves as a basis for identifying and analyzing tasks related to induction, experimentation and intuition in school textbooks. These textbooks, which “constitute an important step in the didactic transposition” (Assude & Margolinas, 2005), represent the institutional interpretation of official requirements. In Tunisia, each level of education has a single textbook, which significantly reflects this interpretation.

2.3.Induction and the Mathematical Knowledge for Teaching (MKT) framework:

To understand how teachers appropriate and implement institutional choices, we draw on the Mathematical Knowledge for Teaching model (Ball et al., 2008). This model distinguishes between two broad categories of knowledge necessary for teaching:

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Table 2: Structure of Mathematical Knowledge for Teaching (MKT) Model. Adapted from Ball et al. (2008),

Subject Matter Knowledge (SMK)	Pedagogical Content Knowledge (PCK)
Common Content Knowledge **(CCK)**: This is the mathematical knowledge and skills used in non-teaching contexts.	Knowledge of Content and Students **(KCS)**: This is the knowledge mathematics and about how students learn and their acquisition processes.
Horizon Content Knowledge **(HCK)**: This is the knowledge necessary to connect content within a mathematical field, such as analysis, and to understand how mathematical topics are related throughout the curriculum.	Knowledge of Content and Teaching **(KCT)**: This is the knowledge that a teacher uses to plan effective teaching and learning situations. It is a combination of expertise in teaching methods and a deep understanding of the mathematical content itself.
Specialized Content Knowledge **(SCK):** This is a type of mathematical knowledge that is specific to the teaching and learning of mathematics. It is not typically needed for purposes other than teaching.	Knowledge of Content and Curriculum **(KCC)**: This is the knowledge about the mathematical content to be taught and how this content is organized in the curriculum.

In addition to these six areas, we add, following Selmi (2024), a seventh area: MMK (Meta-Mathematical Knowledge). This refers to knowledge about the nature of mathematics, its historical and epistemological foundations (Robert & Robinet, 1993). The extended model, inspired by Ball (2008) and Herbst & Kosko (2014), will be referred to by the acronym MKT-AC (A for real analysis as a field of mathematics, C for the continuity of real functions).

In our study, we have chosen to follow Ball's model, described as static, which involves studying teachers' knowledge outside the classroom context. This contrasts with another class of research that follows a model described as dynamic: research that studies teachers' knowledge in the classroom context (Depaepe et al., 2013).

This framework will allow us to interpret teachers' statements not as simple methodological choices, but as indicators of their personal teaching praxeologies in relation to the inductive approach.

3. Methodology

In this work, we draw on the conceptual frameworks proposed by Ball (2008), Bosch and Gascon (2004) and Chevallard (1999).

Initially, the analysis will be conducted using the framework of Anthropological Theory of Didactics (TAD). In this theory, knowledge is present in an institution in the form of praxeologies (Chevallard, 1999; Bosch & Gascon, 2004), modeled as a quadruplet $[T, \tau, \theta, \Theta]$ where: T denotes a type of task; τ denotes a technique for performing these tasks; θ denotes technology (a justificatory discourse on the technique); Θ refers to theory (the broader framework that underpins and validates technological discourse).

This framework will provide the means to examine the status and precise function of inductive reasoning within the praxeological organization (Chevallard, 1999) relating to the introduction of the concept of continuity.

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Secondly, this article aims to analyse the types of knowledge used by certain secondary school mathematics teachers in Tunisia when teaching the concept of continuity of a real function. Using a questionnaire developed for this purpose, it aims to explore representations, teaching practices and identified gaps in order to better identify training needs and identify ways to improve the quality of knowledge transfer.

The development and validation of this questionnaire followed a rigorous process to ensure the validity of its content. An initial version comprising 26 questions was submitted to an expert review panel composed of experienced secondary school teachers, a mathematics education specialist (inspector and researcher in mathematics education) and researchers in mathematics education. Their feedback, focusing on the clarity, relevance, and feasibility of the instrument, led a substantial revision. This process resulted in a final questionnaire of 19 questions, which was submitted to the 67 participants. As part of this article on the inductive method, a targeted thematic analysis was conducted on a subset of four questions dealing directly with the inductive method and its practices.

The analysis is based on work in mathematics education, notably that of Ball et al. (2008), Clivaz (2011) and Selmi (2025), who propose interesting theoretical frameworks for shedding light on teachers' professional knowledge.

4. Results

4.1. Case study: Analysis of an introductory activity

4.1.1. The continuity

The study will focus on activities introducing the concept of continuity in textbooks for third-year secondary school students (aged 17 to 18) in both mathematics and experimental sciences, which are identical for both levels.

In mathematics education, a mathematical definition activity is a series of instructions (tasks T) that lead the student to observe specific cases, identify patterns and formulate a conjecture.

In TAD, induction then functions as an intermediate technology (θ) between concrete tasks of graphical or numerical exploration (T, τ) and the target theory (Θ), which is the formal definition in ε-δ. The central question is: does this technology allow for a progressive and coherent construction of the theory, or does it remain an intuitive discourse disconnected from formalization?

Below is the introductory activity 1, p. 23 that we have translated from French:

Let be the function defined for all real numbers x by: $f(x) = \begin{cases} 2x+1 & \text{if } x > 1 \\ 3 & \text{if } x = 1 \\ 3x & \text{if } x < 1 \end{cases}$

1. Graph the function in the plane with an orthonormal coordinate system $(O, \vec{i}, \vec{j})$.
 a. On the vertical axis, mark the set of real numbers y such that $|y - 3| < 0.1$
 b. From the graph, deduce a sufficient condition on x for $|f(x) - 3| < 0.1$ to be true.
2. Graphically determine a sufficient condition on x for $|f(x) - 3| < 0.01$ to be verified.

Figure 1: Activity 1, p. 23 (translated from French)

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The praxeological analysis of the introductory activity proposed by the textbook highlights a three-step inductive process:

- Graphical observation of specific numerical cases (e.g., for β=0.1; 0.01...),
- Perception of regularities,
- Attempt at generalization.

The praxeological organization of this activity below sheds light on the role of induction in managing the transition from experimentation to formal theory:

Table 3: Praxeology around the formal definition

Praxeological component	Activity content	Role of induction
Tasks (T)	**T1:** Graph the function f defined by pieces **T2:** Graphically represent the set of real numbers y such that $\lvert y-f(1)\rvert \prec \beta$; $(\beta=0,1)$ **T3:** Graphically determine a sufficient condition on x such that $\lvert f(x)-f(1)\rvert \prec \beta$ ($\beta=0,1$ and $\beta=0,01$)	The tasks are specific and particular : $\beta=0,1$ and $\beta=0,01$
Techniques (τ)	**For T1:** Plot each of the restrictions of the function f on the same coordinate system. **For T2:** Draw a horizontal line whose ordinates of the points are in the interval $]f(1)-\beta, f(1)+\beta[$ on the graph. **For T3:** Graphically identify all x-coordinates such that the points $(x, f(x))$ lie within the plotted band.	Graphic and visual techniques, rooted in the specific case.
Technology (θ)	For f (x) to be close to f (1), x must be within a certain range **Inductive reasoning**: generalization based on a specific case	Induction is the intuitive technology that allows a zero definition to emerge from observation.
Theory (Θ)	Definition in b - a : For all $\beta \succ 0$, there exists $\alpha \succ 0$ such as : $\lvert x-a\rvert \prec \alpha \Rightarrow \lvert f(x)-f(a)\rvert \prec \beta.$	Absent from the Activity. The link between inductive technology (θ) and theory (Θ) is not established.

Note:

The analysis reveals that the activity seems to construct an intuitive technology (θ): the idea that proximity can be guaranteed as soon as x is within an appropriate interval by the following statement, which acts as an intermediate (intuitive-kinematic) or 'zero-definition' definition:

The previous activity suggests that it can be made as close to as soon as x is sufficiently close to 1. We then say that the function f is continuous at 1.

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This discourse resembles a conjecture that clearly emerges from an inductive process rooted in the observation of specific cases ($\beta = 0{,}1$ and $\beta = 0{,}01$).

However, there is a clear break: the activity stops at these two specific observations. The crucial phase of generalization, i.e. the transition from 'for $\beta = 0{,}1$ or, for $\beta = 0{,}01$, there exists an interval centered at 1' to 'for all $\beta > 0$, there exists $\alpha > 0$ ' and formalization in the language of quantifiers, is not addressed. The leap from intuitive technology (θ) to theory (Θ): the formal definition is not constructed; it is simply announced (or even imposed) following this 'zero definition' without any explicit link.

This formal definition is following:

> **Definition**
> Let f be a function defined on an open interval I, and let a be a real number in I.
> We say that f is continuous at a if, for any number $\beta > 0$, there exists a number $\alpha > 0$ such that, if x is in I and $|x - a| < \alpha$ then $|f(x) - f(a)| < \beta$.

Figure 2: Formal definition of the manual, p. 23 (translated from French)

4.1.2. The discontinuity :

> The plane is equipped with an orthonormal coordinate system $(O, \vec{i}, \vec{j})$.
>
> We have represented the function f defined by: $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$
>
> 1. Reproduce the figure
> 2. Compute $|f(x) - f(0)|$
> 3. Can we make the quantity $|f(x) - f(0)|$ as small as we want by bringing x close to 0 ?

Figure 3: Activity 2, p. 23 (translated from French)

Then the authors of the textbooks follow with a sentence: "The previous activity illustrates the case of a function that is not continuous at 0."

This Activity, which appears to be an application of the formal definition to the sign function in order to conclude that there is a discontinuity at 0, confirms the incompleteness noted in Activity 1. Although it uses the formal definition, it does not exploit the inductive potential to help students understand the negation of the definition (the existence of a problematic real number $\beta > 0$). In fact, this negation uses a technique that is not available to students: $\exists \beta > 0; \forall \alpha > 0, \exists x \in I; \left(|x - 0| < \alpha \text{ et } |f(x) - f(0)| \geq \beta\right)$.

The treatment of this activity is syntactic; however, without revisiting the reasoning that could lead from the graphical observation of a 'break' to its formal characterization, it remains incomplete. Indeed, for question 2, the student is required to perform a simple calculation (algorithmic/syntactic), but for question 3, they will make an intuitive interpretation in the absence of any formalism, and their answer will probably be no, because as soon as x is not equal to 0, $|f(x) - f(0)| = 1$ it doesn't become small. This reasoning is empirical and is unstructured proof. However, the activity presents a counterexample which, according formal

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definition' rigor of continuity, requires consistent syntactic treatment, which could never be achieved due to the lack of learning about negation of logical implications (the formal definition is a logical equivalence between the defined and the defining).

4.2.Questionnaire results: Teachers' reported practices

The questionnaire, which is designed to be anonymous, aims to gather information on the professional experience of secondary school mathematics teachers and their understanding of the concept of continuity of a function.

The tool comprises 19 main questions (Selmi, 2024), some of which are accompanied by sub-questions requiring reasoned justifications. Aware of the level of difficulty required, we chose not to set any time limits in order to allow teachers to formulate thoughtful responses.

In the context of this article, the search for the types of knowledge we are targeting in teachers is based on four questions from this questionnaire administered to 67 teachers with an average of 28 years of teaching experience who are responsible for teaching Year 12 and Year 13 mathematics and experimental science classes. The areas of knowledge targeted by the four items (Q9, Q10, Q11 and Q19, see Appendices) in the questionnaire are KCC, KCT, KCS, SCK and MMK, as shown in the table below, followed by an analysis of the results for each question:

Below are the questions and the MKT knowledge targeted:

Table 4: Questions and MKT domains targeted

Items	MKT domains
Q9	SCK, MMK
Q10	KCS
Q11	KCT
Q19	KCT, KCS, SCK, MMK

Analysis of responses to question 9:

This question (Appendix 1) compares two approaches to introduce the formal definition: one inductive (based on students' graphical representations, comments on curves, then moving on to formalization). It reveals that the practices reported are favoring induction through observation and concrete examples. The knowledge targeted is SCK.

The results relating to teachers' responses are summarized below:

Table 5: Preferred pedagogical approach

		Frequency	Percentage	Valid	Cumulative
Valid	E1 approach	19	28.4	29.2	29.2
	E2 approach	40	59.7	61.5	90.8
	Another point of view	6	9.0	9.2	100.0
	Total	65	97.0	100.0	
Missed	System	2	3.0		

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Total	67	100.0		

Of the 67 teachers surveyed (97% valid responses): 61.5% prefer the E2 approach, which is inductive: based on graphical representations to develop an intuitive sense of continuity (curve without jumps) before formalization, compared to 29.2% who choose E1, which is transmissive : direct formal presentation of definitions. The others (9.2%) opted for a different point of view. These results reveal a marked preference for the inductive method, taking into account the learning needs of students around the concept of continuity of a function.

Interpretation of choices in terms of MKT (Mathematical Knowledge for Teaching)

- SCK (Specialized Content Knowledge): Teachers favoring E2 mobilise specialized knowledge by linking specific cases (graphs) to general properties through inductive reasoning, promoting a gradual construction of the concept of continuity.
- KCT (Knowledge of Content and Teaching): 59.7% (E2) demonstrate mastery of effective teaching strategies, prioritizing graphical intuition over formal definition, in line with the curriculum (formalization takes a back seat, explained by examples).
- KCS (Knowledge of Content and Students): These teachers anticipate student difficulties (barriers to formal understanding) and adapt their teaching, unlike the 28.4% who chose the E1 approach, adopting a traditional approach that limits engagement and deep understanding.
- KCC (Knowledge of Content and Curriculum): Evident in the justifications (e.g. E22: link to the chapter 'Generalities about functions' and graphs; E21: role of graphs for communication and sharing ideas in class.
- MMK (Meta-Mathematical Knowledge): The 9% with other points of view reflect critical thinking about teaching situations, assessing the impact on conceptual understanding and exploring innovative approaches.

Justifications for Choices (74.2% provided, 40 valid out of 49):

To further refine our analyses, we will examine the valid explanations provided by teachers referring to semiotic representation registers (Duval, 1993, 2017):

Table 6: Distribution of justifications according to representation registers

Criteria	Graphic register	Formal register	Mixed
Effective	16	4	20

Interpretation in terms of MKT knowledge:

The teachers' justifications indicate the use of SCK/KCC knowledge to justify the inductive strategy. However, the lack of justification among some teachers (25.8%) indicates a need for training to develop meta-mathematical knowledge (MMK).

Conclusion of the analysis of question 9:

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The teachers demonstrate strong MKT in KCT/KCS around the inductive approach in teaching continuity, aligning their practices with the needs of students. However, the existence of a minority who chose E1 highlights gaps in KCS and MMK.

Analysis of responses to question 10:

This question (Appendix 2) lists learning difficulties related to concepts such as generalization, counterexamples and graphs. Generalization is a stage of induction (moving from the concrete to the formal), and this takes into account the obstacles perceived by teachers in the inductive construction of the epsilon-delta definition. The results relating to teachers' responses are summarized below:

Table 7: Descriptive statistics on concepts that pose difficulties for teaching continuity

	N	Minimum	Maximum	Average	Standard deviation
Inequalities in IR	64	0	1	.58	.498
The absolute value of a real number	64	0	1	.56	.500
Approximation	64	0	1	.41	.495
Generalization	64	0	1	.38	.488
The use of counterexamples	64	0	1	.44	.500
The use of graphs	64	0	1	.28	.453
N valid (list)	64				

A selective reading of this table allows us to deduce that:

Understanding of approximation (average = 0.41) is perceived as weak. Students' difficulties in grasping this concept can directly influence their ability to understand continuity because it is impossible to implement any inductive process.

Generalization (average = 0.38) is considered to be even less well understood. This can pose a major obstacle for students, as the ability to generalize mathematical properties from specific cases is crucial to understanding more advanced concepts such as continuity, since generalization is the ability to make conjectures.

The very low average for the use of graphs (average = 0.28) indicates a marked difficulty for students in interpreting function graphs. Given that continuity is often visualized graphically, this shortcoming could seriously hinder their understanding of the concept of continuity.

The existence of other concepts that cause difficulties in teaching continuity should not be overlooked. Teachers mention a few other concepts that cause difficulties for students in teaching continuity, such as formalization, and this is due to the lack of teaching of logic, as explained by teacher E48 (each teacher is designated by Ei with “i” is an integer between 1 and 67 and Q-Ei their answer to the question):

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	English translation
Si, vous estimez qu'elles sont liées à d'autres notions, **veuillez les expliciter** : la formalisation : l'absence de l'enseignement de la logique mathématique posent des difficultés persistantes chez les élèves pour comprendre la définition formelle.	"The formalization: the absence of teaching in mathematical logic causes persistent difficulties for students in understanding formal definitions"

Figure 4: Extract from the Q-E48 questionnaire

These results indicate that, according to teachers' statements, the cognitive prerequisites for successfully completing an inductive process are fragile among students.

Analysis of responses to question 11:

This question (Appendix 3) assesses whether the activities in the textbook help to construct the formal definition. These activities are often inductive (explorations, examples), providing data on how teachers perceive their role in a process leading to formalization.

The results are given in the following table:

Table 8: Statements explained: the activities in the textbook could help establish the formal definition of continuity.

		Frequency	Percentag	Valid	Cumulative percentage
Valid	No	13	19.4	19.7	19.7
	Yes	53	79.1	80.3	100.0
	Total	66	98.5	100.0	
Missed	System	1	1.5		
	Total	67	100.0		

According to this table, 53 teachers, or 79.1% of respondents, believe that the activities in the textbook could help to construct the formal definition (in alpha-beta), while 13 teachers, or 19.4% of respondents, believe that the activities in the textbook do not allow for the construction of the formal definition.

Interpretation:

We note that a significant majority of teachers recognize that the activities proposed in the textbook are useful in helping students understand the formal definition of continuity. This indicates confidence in the textbook and also shows that teachers perceive the activities in the textbook as relevant to teaching the concept of continuity of a real function on a real number. This shows that these teachers are in line with the curriculum. However, the other category deserves special attention. Indeed, the explanations provided by some of them help to better understand the issues at stake. Here, for example, is the response from teacher E20:

	English translation
Non, l'élevé est incapable ni d'assimiler la situation ni de la généraliser.	"No, the pupil is unable to comprehend the situation or generalise it."

Figure 5: Extract from the Q-E20 questionnaire

This teacher's response reflects a significant understanding of students and their difficulties, as he justifies his answer with an argument relating to students' cognitive abilities to generalize.

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Conclusion of the analysis of question Q11:

Analysis of the responses to question 11 reveals a predominantly positive perception among teachers regarding the usefulness of the activities in the textbook for constructing the formal definition of the continuity of a function. With 79.1% of teachers stating that these activities can help with this construction, it seems that the textbook is considered a relevant and effective tool for teaching the concept of continuity, and that the introductory activity for the definition meets their expectations. However, the varied responses also show a diversity of views and levels of understanding among teachers. Some, such as teacher E20, seem to adopt a more intuitive approach that is less focused on formalization due to the difficulties associated with generalization and therefore with the inductive process.

Our analysis thus highlights not only teachers' confidence in the teaching resources, but also the richness of their reflections on methods of teaching continuity. However, a significant proportion of teachers is sceptical about the proposed approach and sees it as a source of difficulty for students.

Analysis of responses to question 19:

This question (Appendix 4) compares two strategies for conjecturing and proving continuity: one inductive (graph first, conjecture, then proof).

It examines teachers' reported practices in order to compare students' intuitive conceptions with formalization. Teachers' choices are summarized as follows:

Table 9: Preferred strategy for proving continuity at 0

		Frequency	Percentag	Valid	Cumulative percentage
Valid	P1	20	29.9	33.9	33.9
	P2	39	58.2	66.1	100.0
	Total	59	88.1	100.0	
Missed	System	1	1.5		
	Total	67	100.0		

Of the 67 teachers surveyed (88.1% valid responses, or 59): 33.9% (20 responses) opted for strategy P1 (formal: ε-δ definition only) compared to 66.1% (39 responses) who chose strategy P2 (inductive: graph first, intuitive conjecture, then formal proof) and 8, or 11.9% of respondents, abstained.

These results show a majority preference for the inductive method, combining visual intuition and formalization to conjecture and prove continuity at 0.

Interpretation in terms of MKT (Mathematical Knowledge for Teaching)

- KCT (Knowledge of Content and Teaching): The 66.1% favoring P2 demonstrate mastery of teaching strategies, integrating multiple registers (graphical for intuition, formal for rigor). This reflects an ability to choose progressive approaches in line with the curriculum, unlike the 33.9% (P1) who favor a traditional transmissive method.

- KCS (Knowledge of Content and Students): Teachers who have chosen the P2 strategy anticipate students' cognitive difficulties (need for visual intuition before abstraction), adapting

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their teaching to overcome obstacles. Those who chose the P1 strategy show confidence in formal direct input, but potentially underestimate students' needs.

- SCK (Specialized Content Knowledge): Teachers who opted for the P2 strategy demonstrate a specialized understanding of representations (visualization for intuition, then demonstration), promoting solid conceptualization. Those who chose the P1 strategy demonstrate formal knowledge, but less nuanced understanding of complex concepts.

Justifications for choices (84.7% provided, i.e. 50 out of 59; 39 valid):

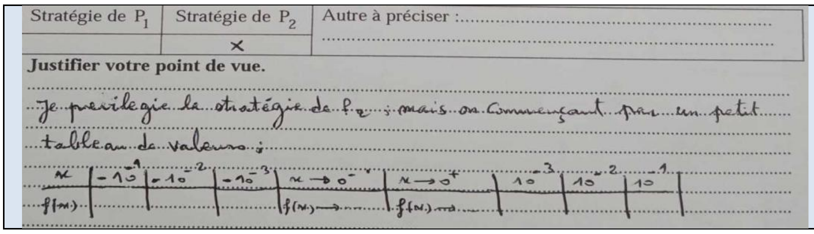
The following table shows the distribution of justifications according to the criteria indicated, which we developed based on the teacher's use of a semiotic representation register (Duval, 1993, 2017):

Table 10: Distribution of justifications according to representation registers

Critères	Graphic	Formal	Graphic-formal articulation	Articulation Table of values-Graph-	Articulation Table of values-Formal	Interactive software
Number	8	15	13	1	1	1

Interpretation in terms of MKT knowledge:

Valid justifications draw on the KCT/KCS/SCK domains (e.g. E11 notes that the graph is insufficient without the formal aspect; E12 articulates the numerical-graphical-formal registers in a clear inductive approach by adding a table of values):



Stratégie de P_1	Stratégie de P_2	Autre à préciser :
	×	

Justifier votre point de vue.

Je privilégie la stratégie de P_2 ; mais on commençant par un petit tableau de valeurs ;

English translation
"I favour the P2 strategy, but starting with a small table of values."

Figure 6: Extract from the Q-E12 questionnaire

Finally, the absence of justification accounts for 15.3%, indicating gaps in teachers' MMK skills (no justification, indicating a lack of critical thinking).

Conclusion of the analysis of Q19:

Teachers demonstrate strong MKT in KCT/KCS/SCK for the inductive approach to continuity, favoring a progression from intuition to formalism adapted to students. The minority ho chose the P1 strategy and the lack of justification (28 out of 67) indicate doubts in MKT-AC, highlighting a need for training to strengthen critical thinking and more engaged attitudes.

5. Discussion

The results of this study highlight a significant gap between curricular intentions and their actual implementation in continuity teaching in Tunisian secondary schools. On the one hand, the praxeological analysis (TAD) of the introductory activity in the textbook reveals that

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induction is used as an intuitive technology (θ), anchored in concrete tasks (T) and graphic techniques (τ), leading to a kinematic 'zero definition'. However, the transition to formal theory (Θ) (the definition in ε-δ) remains difficult, without explicit generalization of particular cases ($\beta = 0.1$; 0.01) to universal quantifiers. This confirms the hypothesis of incomplete induction, which fails to serve as a mediator, as suggested by Lakatos (1984) with his zero-definitions. For discontinuity, the absence of inductive treatment of negation reinforces this break, highlighting a lack of teaching of formal logic, which is essential for manipulating counterexamples.

On the other hand, teachers' responses to the questionnaire indicate that induction is widely adopted: 61.5% prefer a graphical-intuitive approach before formalization (Q9), and 66.1% opt for a strategy combining visual conjecture and ε-δ proof (Q19). These choices draw on solid MKT domains, particularly KCT (progressive teaching strategies) and KCS (anticipating student obstacles, such as generalization, which is perceived as weak by 38% in Q10). Nevertheless, a minority (29.2% in Q9; 33.9% in Q19) favors' direct transmission, revealing gaps in MMK (meta-mathematical thinking) and an underestimation of students' cognitive needs. Confidence in the textbook activities (79.1% in Q11) contrasts with criticism of students' inability to generalize, showing that resources, although they accurately reflect the curriculum, are not sufficient without reasoned and critical use by expert teachers.

These results are part of a broader context of transition from secondary school to university (Artigue, 2004; Ghedamsi, 2008), where incomplete induction contributes to failure at university. Compared to other countries (France, Vietnam), Tunisia introduces the formal definition of continuity with an inductive approach, but without systematic deductive validation, which limits its effectiveness. The obstacles identified (approximation at 41%, graphs at 28% in Q10) point to unmastered prerequisites, calling for the integration of logic (Soltani & Chellougui, 2025) and digital tools into teaching. Furthermore, the adoption of the MMK domain enriches the MKT model by emphasizing the need for teachers to master the epistemological nature of induction, particularly the distinction between complete and incomplete induction (Soltani & Chellougui, 2024), in order to make it a relevant approach in the construction of rigorous knowledge.

6. Conclusion

This research demonstrates that induction, although prescribed as an essential lever in Tunisian programmes, often remains incomplete in the construction of the concept of continuity, leading to breaks between intuition and formalization. Cross-analysis using TAD and MKT-AC tools reveals that resources (programmes and textbooks) and teaching practices encourage graphic exploration but struggle to ensure generalization and rigorous validation, contributing to the difficulties observed at university. To optimize its role, it is recommended to: (1) enrich the textbooks activities from a didactic perspective of constructing definitions initiated by Ouvrier-Buffet (2013), through explicit stages of generalization (from the particular to the quantified universal) and by integrating structured teaching of negation to characterize discontinuity; (2) Strengthen teacher training on the epistemological dimension of formalism and induction (the MMK domain) and on the articulation between intuitive and formal registers so that they can effectively manage the transition from 'zero definition' to rigorous definition; (3) Use digital tools (dynamic visualization, simulations) to encourage inductive experimentation and offer a concrete perception of proximity concepts (epsilon-delta).

This study is an educational contribution linking the TAD and MKT frameworks to diagnose, in terms of teaching resources and knowledge, the phenomenon of incomplete or even

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‘truncated’ induction. It paves the way for classroom observations to analyse how this truncated induction is actualized or overcome in teacher-student interaction, and to design targeted interventions.

References

Artigue, M. (1999). The teaching and learning of mathematics at the university level: Crucial questions for contemporary research in education. Notices of the American Mathematical Society, 46(11), 1377–1385 . https://www.ams.org/notices/199911/fea-artigue.pdf

Artigue, M. (2004). Le défi de la transition secondaire–université : Que peuvent nous apporter les recherches et les innovations développées dans ce domaine ? In Actes du premier congrès franco-canadien des sciences mathématiques. Toulouse, France.

Assude, T., & Margolinas, C. (2005). Aperçu sur les rôles des manuels dans les recherches en didactique des mathématiques. In É. Bruillard (Ed.), Manuels scolaires, regards croisés (pp. 231–241). SCÉRÉN–CRDP de Basse-Normandie.https://shs.hal.science/hal-00779302

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? Journal of Teacher Education, 59(5), 389–407 . https://doi.org/10.1177/0022487108324554

Bloch, I. (2000). L’enseignement de l’analyse à la charnière lycée–université : Savoirs, connaissances et conditions relatives à la validation [Doctoral dissertation, Université Bordeaux I]. HAL Thèses. https://theses.hal.science/tel-01222400

Bosch, M., & Gascón, J. (2004). La praxéologie comme unité d’analyse des processus didactiques. In C. Margolinas, M. Abboud-Blanchard, L. Bueno-Ravel, N. Douek, A. Flückiger, P. Gibel, F. Vandebrouck, and F. Wozniak (Eds.), Balises pour la didactique des mathématiques (pp. 107–122). La Pensée Sauvage.

Centre National Pédagogique. (2007). Manuel de mathématiques : Troisième année de l’enseignement secondaire, section mathématiques (Tome 1). Ministère de l’Éducation, République Tunisienne.

Chevallard, Y. (1999). L’analyse des pratiques enseignantes en théorie anthropologique du didactique. Recherches en Didactique des Mathématiques, 19(2), 221–266.

Clivaz, S. (2011). Des mathématiques pour enseigner : Analyse de l’influence des connaissances mathématiques d’enseignants vaudois sur leur enseignement des mathématiques à l’école primaire [Doctoral dissertation, Université de Genève]. Archive ouverte UNIGE.

Doi: https://doi.org/10.61856/4xqe5d92

https://archive-ouverte.unige.ch/unige:17047

Crocco, G., Audureau, É., & Michel, A. (2020). Édition commentée et annotée de Henri Poincaré, La science et l'hypothèse : Préface, chapitres 1 et 2. HAL . https://hal.science/halshs-02494965

Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. Teaching and Teacher Education, 34, 12–25 . https://doi.org/10.1016/j.tate.2013.03.001

Direction de la Pédagogie et des Normes. (2008). Programmes de mathématiques : Enseignement secondaire. Ministère de l'Éducation et de la Formation, République Tunisienne.

Duval, R. (1993). Registres de représentation sémiotique et fonctionnement cognitif de la pensée. Annales de Didactique et de Sciences Cognitives, 5, 37–65.

Duval, R. (2017). Understanding the mathematical way of thinking: The registers of semiotic representations (T. M. M. Campos, Ed.). Springer .https://doi.org/10.1007/978-3-319-56910-9

El Bouazzaoui, H. (1988). Conceptions des élèves et des professeurs à propos de la notion de continuité d'une fonction [Doctoral dissertation, Université Laval].

Ghedamsi, I. (2008). Enseignement du début de l'analyse réelle à l'entrée à l'université : Articuler contrôles pragmatique et formel dans des situations à dimension adidactique [Doctoral dissertation, Université de Tunis and Université Victor Segalen Bordeaux II]. HAL Thèses.

https://theses.hal.science/tel-00361848

Hasibah, B., Rohaeti, E. E., & Aryan, B. (2018). Application of inductive-deductive approach to improve the ability of mathematical communication and self-efficacy of junior high school students. Journal of Innovative Mathematics Learning, 1(2), 70–75 . https://doi.org/10.22460/jiml.v1i2.p70-75

Herbst, P., & Kosko, K. W. (2014). Mathematical knowledge for teaching and its specificity to high school geometry instruction. In J.-J. Lo, K. R. Leatham, & L. R. Van Zoest (Eds.), Research trends in mathematics teacher education (pp. 23–45). Springer.

https://doi.org/10.1007/978-3-319-02562-9_2

Jeannotte, D. (2015). Raisonnement mathématique : Proposition d'un modèle conceptuel pour l'apprentissage et l'enseignement au primaire et au secondaire [Doctoral dissertation, Université du Québec à Montréal]. Archipel . https://archipel.uqam.ca/8129

Doi: https://doi.org/10.61856/4xqe5d92
Lakatos, I. (1984). Preuves et réfutations : Essai sur la logique de la découverte mathématique. Hermann.

Binti Misrom, N. S., Muhammad, A. S., Abdullah, A. H., Osman, S., Hamzah, M. H., & Fauzan, A. (2020). Enhancing students' higher-order thinking skills through an inductive reasoning strategy using GeoGebra. International Journal of Emerging Technologies in Learning, 15(3), 156–179 .https://doi.org/10.3991/ijet.v15i03.9839

Ouvrier-Buffet, C. (2013). Modélisation de l'activité de définition en mathématiques et de sa dialectique avec la preuve : Étude épistémologique et enjeux didactiques [Habilitation à diriger des recherches, Université Paris-Diderot–Paris VII]. HAL Thèses .https://theses.hal.science/tel-00964093

Poincaré, H. (1902). La science et l'hypothèse. Flammarion.

Pólya, G. (1954). Mathematics and plausible reasoning: Vol. 1. Induction and analogy in mathematics. Princeton University Press .https://doi.org/10.2307/j.ctv14164db

Pólya, G. (1967). La découverte des mathématiques. Dunod.

Purnawasi, M. (2024). The effectiveness of inductive teaching in mathematics [Master's thesis, West Texas A&M University]. WTAMU Institutional Repository . https://wtamu-ir.tdl.org/items/9ffead3d-cbac-4d78-96f8-5fdb5b2c67be

Robert, A., & Robinet, J. (1993). Prise en compte du méta en didactique des mathématiques (Cahier de DIDIREM No. 21). IREM de Paris .https://hal.science/hal-02140979

Selmi, A. (2024). De la topologie à l'analyse réelle au lycée : Effets de transposition. Étude croisée en didactique et épistémologie des mathématiques : Cas de la notion de continuité d'une fonction dans l'enseignement secondaire tunisien [Unpublished doctoral dissertation, Université Virtuelle de Tunis].

Selmi, A. (2025). Les défis des définitions formelles de la continuité des fonctions réelles dans l'enseignement secondaire tunisien. Journal Wisdom pour les Études et la Recherche, 5(4), 361–380 .https://doi.org/10.55165/wjfsar.v5i04.675

Sghaier, S. B. (2019). Ingénierie d'intégration des TIC dans l'enseignement du concept de continuité dans le cycle secondaire tunisien [Doctoral dissertation, Sorbonne Université and Université Virtuelle de Tunis]. HAL Thèses . https://theses.hal.science/tel-03026149

Singh, N. K., & Yadav, A. K. (2017). Inductive and deductive methods in mathematics teaching. International Journal of Engineering Research and Applications, 7(11, Part 2), 19–22 .https://doi.org/10.9790/9622-0711021922

Soltani, W., & Chellougui, F. (2024). Inductive reasoning: Problems, methods of justification and interaction between mathematics and computer science. The International Innovations Journal of Applied Science, 1 .(2)https://doi.org/10.61856/095nzv52

Doi: https://doi.org/10.61856/4xqe5d92

Soltani, W., & Chellougui, F. (2025). Study of the internal didactic transposition of the concept of reasoning by mathematical induction in Tunisian secondary education. African Journal of Advanced Studies in Humanities and Social Sciences, 1(5), 1–10 .https://doi.org/10.65418/ajashss.v1i5.1697

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. Educational Studies in Mathematics, 12(2), 151–169 .https://doi.org/10.1007/BF00305619

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Appendices

Appendix 1: Question 9 (translated from French)

Here are two approaches to introducing the formal definition of the concept of continuity of a function at a real number by two teachers E1 and E2:

E1 approach	**E2 approach**
-The teacher E1 writes on the board the definition proposed by the textbook for the continuity of a function f at a real number a. -E1 draws the graph of an arbitrary function, -E1 chooses an interval centered at a on the x-axis. -E1 determines the corresponding interval on the y-axis centered on f (a). -E1 presents examples that he or she deals with using the formal definition.	-The teacher E2 asks the students to make graphical representations of certain common functions. -E2 asks the students to comment on the curve representing each function. -E2 introduces the formal definition from the textbook. -E2 discusses the previous examples using this formal definition.

In your opinion, which of these two approaches is the most relevant?

E1 approach	E2 approach	Another point of view

Please explain your choice:

……..……………………………………………………………………

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Appendix 2: Question 10 (translated from French)

In your opinion, do students encounter difficulties in learning the concept of continuity of a function at a real number related to other concepts?

Please indicate them in the table below:

Inequalities in IR	
The absolute value of a real number	
Approximation	
Generalization	
Use of counterexamples	
Use of graphs	

If you believe they are related to other concepts, please explain: ……………………………..

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Appendix 3: Question 11 (translated from French)

Do you think that the activities in the textbook could help to construct the formal definition (in alpha-beta) of the continuity of a function on a real number?

Explain:

…………………………………………………………………………………………..

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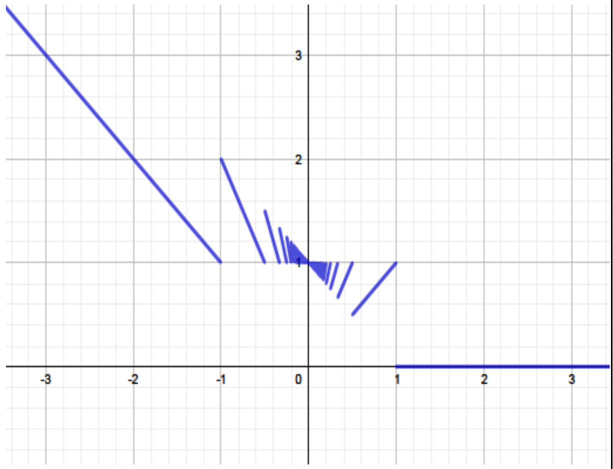
Appendix 4: Question 19 (translated from French)

Consider the function f defined on IR by : $f(x) = x \times \text{floor}\left(\frac{1}{x}\right)$ if $x \neq 0$ and $f(0) = 1$.

Two teachers, P1 and P2, ask final-year students (mathematics and experimental science sections) to formulate a conjecture about the continuity of the function f at 0 and to prove this conjecture. The students produce three types of answers: f is continuous at 0; f is discontinuous at 0; it is impossible to know.

To help the students resolve this situation, each teacher developed a strategy:

P1 strategy	**P2 strategy**
-P1 asks his students to produce formal proof of their conjecture. -After several attempts, P1 writes the following proof on the board: Let $\varepsilon > 0$. Is there an $\delta > 0$ such that: If $x \in$ IR and $\lvert x-0 \rvert < \delta$ then $\lvert f(x)-f(0) \rvert < \varepsilon$? For all $x \neq 0$, we have : $\left\lfloor \frac{1}{x} \right\rfloor \le \frac{1}{x} < \left\lfloor \frac{1}{x} \right\rfloor + 1$ $\Rightarrow 0 \le \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor < +1 \Rightarrow \left\lvert \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor \right\rvert < 1$ By multiplying both sides by $\lvert x \rvert$, we obtain**:** $\Rightarrow \lvert x \rvert \left\lvert \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor \right\rvert < \lvert x \rvert \Rightarrow \left\lvert 1 - x.\left\lfloor \frac{1}{x} \right\rfloor \right\rvert < \lvert x \rvert$ $\Rightarrow \left\lvert x.\left\lfloor \frac{1}{x} \right\rfloor - 1 \right\rvert < \lvert x \rvert \Rightarrow \lvert f(x) - f(0) \rvert < \lvert x \rvert$ Thus, $x \in IR, \lvert x \rvert < \delta \Rightarrow \lvert f(x) - f(0) \rvert < \lvert x \rvert < \varepsilon$ Just take $d = e$ and then f is continuous at 0.	-P2 provides a paper template on which the curve of the function is plotted:  -He asks them to make a conjecture based on the figure. -He invites the students who have stated that the function f is continuous at 0 to produce a formal proof of their conjecture. -After some attempts, he writes a proof on the board similar to the one written by the teacher P1.

If you were in the shoes of these two teachers, which of the two strategies would you choose to address the situation presented to the students? :

P1 strategy	**P2 strategy**	**Other to be specified**: ....
		....

Justify your point of view: ....